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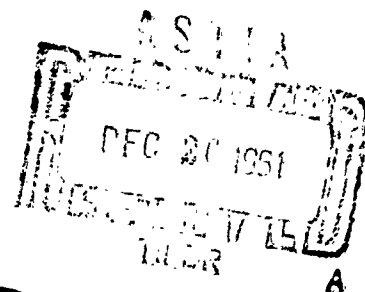
UNIVERSITY OF SOUTHERN CALIFORNIA

SCHOOL OF ENGINEERING

Technical Note

ON TRANSPORT PROCESSES IN A PLASMA (KINETIC EQUATION FOR A PLASMA)

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Engineering Center

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(KINETIC EQUATION FOR A PLASMA)

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ABSTRACT

Authors usually neglect the effect of Coulomb field in the space of radius $e^2/(3kT/2)$ surrounding a charged particle, and derive equations of the Fokker-Planck type from the Liouville equation of a plasma. ~~In this paper,~~ the effect of the inner core field, neglected usually, is shown to be present in an equation of the Fokker-Planck type as an additional term which is similar to the collision term in an equation of the Boltzmann type. It is shown that the order of this additional term may easily be larger than those of the friction and diffusion terms, the effect of the outer core field. When the effect of the outer core field is larger than that of the inner core field, the assumption of binary interaction is shown to be not feasible. The possible nonlinear and non-Markovian behavior of a particle is neglected.

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LIST OF MAIN SYMBOLS

\vec{A}	friction
\vec{B}	diffusion
e_a	electric charge of particle a
E_a	macroscopic electric force on particle a
F_a, F_b, F_c	one-particle distribution functions
F_{ab}, F_{ac}	two-particle distribution functions
F'_{ab}	$F_{ab} - F_a F_b$
L_D	Debye length (the radius of the outer core field) [see Eq. (2.4)]
m_a	mass of particle a
N_a	total number of particle a in volume V
n_a	N_a/V
\vec{p}_a	momentum of particle a
\vec{q}_a	position vector of particle a
R	the radius of the inner core field [see Eq. (2.5)]
$\left. \begin{matrix} s \\ t \end{matrix} \right\}$	time
V	volume of the container
δ	$n^{-\frac{1}{3}}$
ϕ_{ab}	potential energy between two particles a and b

ϕ_{ab_1}	potential outside the outer core
$\phi_{ab_{II}}$	potential inside the outer core
$\phi_{ab_{III}}$	potential inside the inner core
τ_{II}	the average period of correlation in the outer core field [see Eq. (3.1)]
τ_{III}	the average period of correlation in the inner core field [see Eq. (3.2)]

I. INTRODUCTION

We investigate the process of deriving the equation of the one-particle distribution function of a plasma and show that it is convenient to give the interaction term as the sum of the term of collision which is characteristic of the Boltzmann equation and those of diffusion and friction which are proper to the Fokker-Planck equation.

Usually authors (Refs. 1, 2 and 3) derive equations of the Fokker-Planck type by assuming that the probability of a pair of particles interacting with each other at distances shorter than $e^2/(3kT/2)$ is negligibly small. The interaction outside this distance (and inside the Debye length distance) is weak and of a fairly long correlation time. Therefore, it is possible to expand the s -body function similarly to the Mayer cluster expansion and the interaction in this outer core is conveniently presented by the terms of friction and of diffusion. Here we pose the question whether it is proper to neglect the effect of the inner core of the field.

According to the result in this paper, the effect of the inner core field may easily be larger than those of the friction and diffusion terms caused by the outer core field. The probability of the inner core correlation is small. But the correlation itself is strong. Thus the total effect may not be negligible.

With respect to the inner core field, usual methods of expanding correlation functions similar to Mayer's cluster expansion may not be readily applied, as the expansion may not converge. In this paper, we present the effect of strong interaction of short correlation periods in the inner core by "collision" and the weak interaction of long correlation periods in the outer core by "friction and diffusion." As a result, the equation of a one-body function includes the "collision" term and "friction and diffusion" terms. Unlike the case of neutral particles, two particles immediately before a "collision" by means of the inner core field are correlated to each other due to the outer core field. We note, however, that weak correlations in the outer core are effective because of long correlation periods. These weak correlations are not effective when we integrate them over short periods of interactions in the inner core. In the short period of interaction in the inner core, only the strong interaction developed in the inner core is effective. Therefore we may neglect the history (or memory) formed in the outer core, when we consider the interaction in the inner core. This is our assumption which makes the present investigation possible. In the future, we may consider the interaction between these two kinds of correlations which are to be one, but now separated rather artificially. Related to this point of the problem, we note that Tchen (Ref. 4) recently investigated the interaction between the correlation caused by the outer

core field, and the effect of the macroscopic field caused by the macroscopic nonuniformity of a plasma. In this paper, these two kinds of interactions are neglected as secondary effects.

II. ASSUMPTION OF BINARY CORRELATIONS

We consider a mixture of particles of different kinds and assume that all the particles are electrically charged and that their interactions are caused by Coulomb electrostatic force. The distribution function of a single particle belonging to a group will be denoted by F_a , F_b , etc. The binary correlation function for a pair of particles will be denoted by F_{ab} , F_{ac} , etc. By integrating the Liouville equation with respect to the position and momentum coordinates (\vec{q}, \vec{p}) of all the particles, except for one, we obtain

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a}\right) F_a - \sum_b \frac{N_b}{V} \int \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial F_{ab}}{\partial \vec{p}_a} d\vec{q}_b d\vec{p}_b = 0 \quad (2.1)$$

Here

$$\phi_{ab} = \frac{e_a e_b}{|\vec{q}_a - \vec{q}_b|} \quad (2.2)$$

and the summation \sum_b is to be extended to all the particles except for the one under consideration. For the equation of F_{ab} , we obtain

$$\begin{aligned}
& \left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} + \frac{\vec{p}_b}{m_b} \cdot \frac{\partial}{\partial \vec{q}_b} - \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial}{\partial \vec{p}_a} - \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial}{\partial \vec{p}_b} \right) F_{ab} \\
& = \sum_c \frac{N_c}{V} \int \left(\frac{\partial \phi_{ac}}{\partial \vec{q}_a} \cdot \frac{\partial F_{abc}}{\partial \vec{p}_a} + \frac{\partial \phi_{bc}}{\partial \vec{q}_b} \cdot \frac{\partial F_{abc}}{\partial \vec{p}_b} \right) d\vec{q}_c d\vec{p}_c
\end{aligned} \tag{2.3}$$

According to the results of Tchen's analysis (Ref. 1), we separate the domain of $|\vec{q}_a - \vec{q}_b|$ in three:

Domain I:

$$|\vec{q}_a - \vec{q}_b| > L_D = \left(\sum_b \frac{4\pi N_b e_b^2}{V k T_0} \right)^{-\frac{1}{2}} \tag{2.4}$$

Domain II (Outer Core):

$$L_D \geq |\vec{q}_a - \vec{q}_b| > R_{ab} = \frac{e^2}{(3kT/2)} \tag{2.5}$$

Domain III (Inner Core):

$$R_{ab} \geq |\vec{q}_a - \vec{q}_b| \tag{2.6}$$

Here $3kT/2$ is the average energy of a particle. In Domain I, we may write for F_{ab} simply

$$F_{ab} = F_a F_b \tag{2.7}$$

The interaction of particles a and b is effective only when the distribution is not spatially uniform. Of course, we neglect the weak

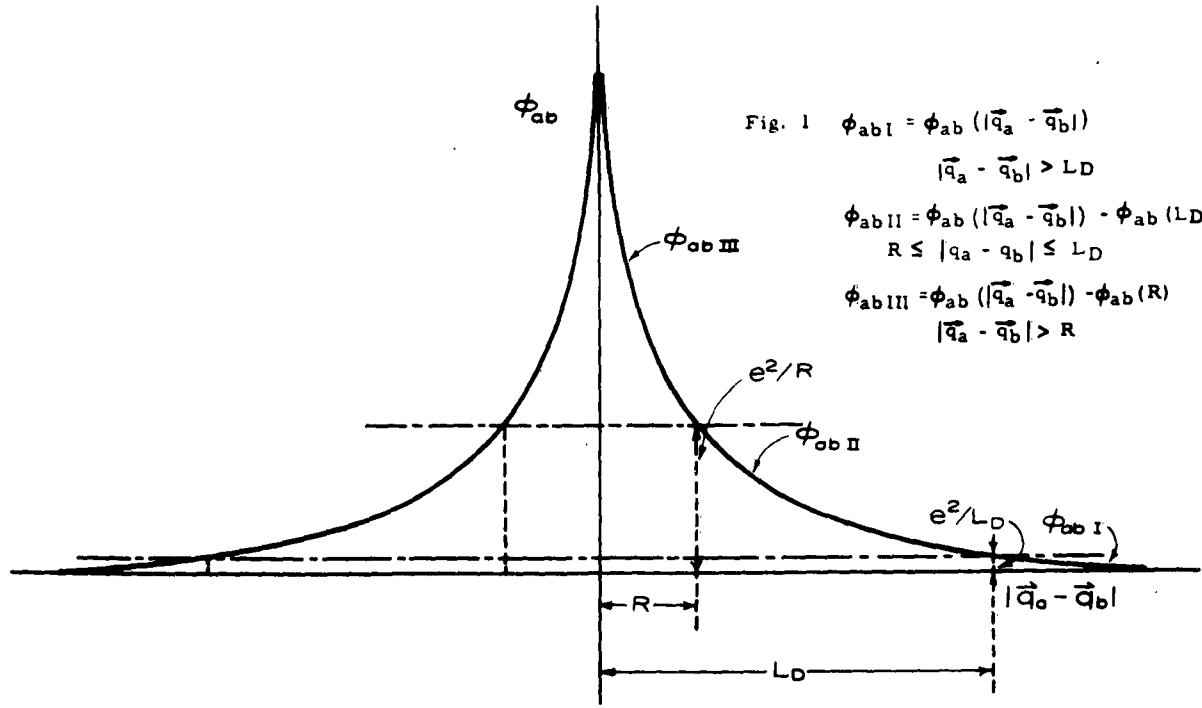
correlation introduced by the spatial nonuniformity which was considered by Tchen recently (Ref. 4). In Domain II, we write for F_{ab}

$$\begin{aligned}
 F_{ab} &= F_a F_b + F_{ab}' \\
 F_{abc} &= F_a F_{bc} + F_b F_{ca} + F_c F_{ab} - 2 F_a F_b F_c \\
 F_{ab}'/F_a F_b &< 1
 \end{aligned}
 \tag{2.8}$$

Because of Eq. (2.5), it is permissible to do so, as carefully studied by Tchen (see Ref. 1, Appendix A). In Domain III, however, any rapid convergence of an expansion similar to Eq. (2.8) may not be expected. Separating the domain of the integration, with respect to \bar{q}_b , we write for Eq. (2.1)

$$\begin{aligned}
 & \left(\frac{\partial}{\partial t} + \frac{\bar{p}_a}{m a} \cdot \frac{\partial}{\partial \bar{q}_a} \right) F_a - \sum_b \frac{N_b}{V} \int_I \frac{\partial \phi_{ab}}{\partial \bar{q}_a} \cdot \frac{\partial F_a F_b}{\partial \bar{p}_a} d\bar{q}_b d\bar{p}_b \\
 & - \sum_b \frac{N_b}{V} \int_{II} \frac{\partial \phi_{ab}}{\partial \bar{q}_a} \cdot \frac{\partial (F_a F_b + F_{ab}')}{\partial \bar{p}_a} d\bar{q}_b d\bar{q}_b \\
 & - \sum_b \frac{N_b}{V} \int_{III} \frac{\partial \phi_{ab}}{\partial \bar{q}_a} \cdot \frac{\partial F_{ab}}{\partial \bar{p}_a} d\bar{q}_b d\bar{p}_b = 0
 \end{aligned}
 \tag{2.9}$$

Instead of separating the domain of integration with respect to \bar{q}_b , it might be convenient to separate the potential ϕ_{ab} into three parts as shown in Fig. 1 (see following page). The singularities at the boundaries of domains are simple and may not cause any serious difficulties in our



mathematical treatment. Concerning Eq.(2.9), we may define the macroscopic electric field by

$$\vec{E}_a = -\frac{1}{e_a} \sum_b \frac{N_b}{V} \int_{I+II} \frac{\partial \phi_{ab}}{\partial \vec{q}_a} F_b d\vec{q}_b d\vec{q}_b \quad (2.10)$$

We write for Eq.(2.9)

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} + e_a \vec{E}_a \cdot \frac{\partial}{\partial \vec{p}_a} \right) F_a \\ &= \sum_b \frac{N_b}{V} \int_{II} \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial F_{ab}}{\partial \vec{p}_a} d\vec{q}_b d\vec{p}_b \\ &+ \sum_b \frac{N_b}{V} \int_{III} \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial F_{ab}}{\partial \vec{p}_a} d\vec{q}_b d\vec{p}_b \end{aligned} \quad (2.11)$$

Our further task is to obtain F'_{ab} in Domain II and F_{ab} in Domain III. The terms on the right-hand side of Eq. (2.3) may be neglected if we assume that the nonlinear correlation is negligibly small. After neglecting these terms, there remains no term of integration with respect to \vec{q}_b and \vec{p}_b . Therefore, it is possible to write Eq. (2.3), separately in Domain II and III, as follows:

In Domain II,

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} - \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial}{\partial \vec{p}_a} - \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial}{\partial \vec{p}_b} \right) (F_a F_b + F'_{ab}) = 0$$

By considering Eq. (2.1), we substitute

$$\sum_b \frac{N_b}{V} \int \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial F_{ab}}{\partial \vec{p}_a} d\vec{q}_b d\vec{p}_b$$

for

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} \right) F_a$$

in the above equation. We neglect the nonlinear terms which appear in the equation after the substitution, and also $\frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial F'_{ab}}{\partial \vec{p}_a} + \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial F'_{ab}}{\partial \vec{p}_b}$ which, according to Tchen's study, is not significant so far as

Domain II is concerned. Therefore, for the equation to which F'_{ab}

is subject, we obtain

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} + \frac{\vec{p}_b}{m_b} \cdot \frac{\partial}{\partial \vec{q}_b} \right) F'_{ab} \\ & = \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial F_a}{\partial \vec{p}_a} F_b + \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial F_b}{\partial \vec{p}_b} F_a \end{aligned} \quad (2.12)$$

Equation (2.12) is valid only in Domain II.

In Domain III, neglecting the nonlinear terms in Eq.(2.3), we have

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} + \frac{\vec{p}_b}{m_b} \cdot \frac{\partial}{\partial \vec{q}_b} - \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial}{\partial \vec{p}_a} - \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial}{\partial \vec{p}_b} \right) F_{ab} = 0 \quad (2.13)$$

III. CORRELATIONS

In Domain II, the average period of correlations is long, although the correlations are weak.

$$\tau_{II} = \frac{L_D}{(3kT/m)^{1/2}} \quad (3.1)$$

gives the order of the average period. In Domain III, the average period of correlations is short although the correlations are strong.

$$\tau_{III} = \frac{R}{(3kT/m)} \quad (3.2)$$

We assume that the ratio of two periods is sufficiently larger than unity.

$$\frac{\tau_{II}}{\tau_{III}} = \frac{L_D}{R} > 1 \quad (3.3)$$

Since the changes of variables in Domain II during the period τ_{II} are assumed to be negligibly small, we obtain

$$\begin{aligned}
 & \left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} + e_a \vec{E}_a \cdot \frac{\partial}{\partial \vec{p}_a} \right) \bar{F}_a \\
 &= \sum_b \frac{N_b}{V} \int_{II} \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial}{\partial \vec{p}_a} \bar{F}_{ab}' dq_b d\rho_b \\
 &+ \frac{1}{\tau_{II}} \int_s \int_{II} \sum_b \frac{N_b}{V} \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial \bar{F}_{ab}}{\partial \vec{p}_a} d\vec{q}_b d\vec{p}_b ds
 \end{aligned} \tag{3.4}$$

by averaging each term of Eq.(2.11) over τ_{II} . Here, dashed characters denote respectively the averages of the original functions over τ_{II} along the trajectories of concerned particles:

$$\bar{F}_a = \frac{1}{\tau_{II}} \int_0^{\tau_{II}} F_a(t+s) ds$$

(i) The last term of the right-hand side of Eq.(3.4) may be calculated by means of Eq.(2.3). Here we simply mention the elaborate method by Kirkwood and Ross (Ref.5). They consider a phase space transformation function $K^{(2)}$. Substituting

$$F_{ab}(q, p; t+s) = \int \kappa^{(2)}(q, p/q', p'; s) F_{ab}(q', p'; t) d\vec{q}' d\vec{p}'$$

for F_{ab} in Eq.(2.3), we obtain the equation of $K^{(2)}$. We do not repeat their investigation in detail. However, we must remember that the necessary assumptions by which the last term of Eq.(3.4) is reduced to the collision integral are not precisely given in our case. The

correlation developed in Domain II does exist before the two particles begin to interact in Domain III. As mentioned previously, however, we assume that the correlation in Domain II is weak and that its integral over τ_{II} is not significant as compared with the integral of the strong correlation newly developed in Domain III. This collision term may be calculated by assuming that the potential between two particles is the inner core potential as given in Fig. 1. The collision term thus obtained is denoted by $(\frac{\partial \bar{F}_a}{\partial t})_{coll.}$

$$(\frac{\partial \bar{F}_a}{\partial t})_{coll.} = \frac{1}{\tau_{II}} \int \int_s \sum \frac{N_b}{V} \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial \bar{F}_{ab}}{\partial \vec{p}_a} d\vec{q}_b d\vec{p}_b ds \quad (3.5)$$

(ii) By averaging each term of Eq.(2.12), over τ_{II} , we obtain

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} + \frac{\vec{p}_b}{m_b} \cdot \frac{\partial}{\partial \vec{q}_b} \right) \bar{F}_{ab} \\ & = \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial \bar{F}_a}{\partial \vec{p}_a} \bar{F}_b + \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial \bar{F}_b}{\partial \vec{p}_b} \bar{F}_a \end{aligned} \quad (3.6)$$

Here, of course, we assume that in Domain II, the changes of functions in τ_{II} are small. By integrating the equation along the trajectories of two particles, we obtain

$$\bar{F}_{ab}'(t) = \int_{-\infty}^t \left[\frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial \bar{F}_a}{\partial \vec{p}_a} \bar{F}_b + \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial \bar{F}_b}{\partial \vec{p}_b} \bar{F}_a \right] ds \quad (3.7)$$

The integrand is considered to be a function of $\vec{q}_a(s)$, $\vec{q}_b(s)$.

$$q_a(s) = q_a(t) - (t-s)p_a/m_a$$

$$q_b(s) = q_b(t) - (t-s)p_b/m_b$$

Since $\frac{\partial \phi_{ab}}{\partial q_a} \cdot \frac{\partial F_{ab}}{\partial p_a} + \frac{\partial \phi_{ba}}{\partial q_b} \cdot \frac{\partial F_{ab}}{\partial p_b}$ is neglected in Domain II, p_a and p_b are considered invariants. Substituting Eq.(3.7), we obtain for the first term of the right-hand side of Eq.(3.4),

$$\begin{aligned} & \sum \frac{N_b}{V} \int_{\Pi} \frac{\partial \phi_{ab}(t)}{\partial \vec{q}_a} \cdot \frac{\partial}{\partial \vec{p}_a} \frac{\partial}{\partial \vec{p}_a} \left[\int_{t-\tau_{II}}^t \frac{\partial \phi_{ab}(s)}{\partial \vec{q}_a} \bar{F}_a \bar{F}_b ds \right] d\vec{q}_b d\vec{p}_b \\ & + \sum_b \frac{N_b}{V} \int_{\Pi} \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot \frac{\partial}{\partial \vec{p}_a} \left[\int_{t-\tau_{II}}^t \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial F_b}{\partial \vec{p}_b} F_a ds \right] d\vec{q}_b d\vec{p}_b \end{aligned}$$

Here $\frac{\partial \phi_{ab}(s)}{\partial \vec{p}_a}$ is neglected in the non-Markovian approximation.

Defining \bar{A} and \bar{B} by

$$-\bar{A} \bar{F}_a(t) = \sum_b \frac{N_b}{V} \int_{\Pi} \frac{\partial \phi_{ab}(t)}{\partial \vec{q}_b} \left[\int_{t-\tau_{II}}^t \frac{\partial \phi_{ba}}{\partial \vec{q}_b} \cdot \frac{\partial F_b}{\partial \vec{p}_b} F_a ds \right] d\vec{q}_b d\vec{p}_b \quad (3.8)$$

$$\bar{B} \bar{F}_a(t) = \sum_b \frac{N_b}{V} \int_{\Pi} \frac{\partial \phi_{ab}(t)}{\partial \vec{q}_a} \left[\int_{t-\tau_{II}}^t \frac{\partial \phi_{ab}}{\partial \vec{q}_a} \cdot F_a F_b ds \right] d\vec{q}_b d\vec{p}_b \quad (3.9)$$

the interaction in Domain II is represented by

$$-\frac{\partial}{\partial \vec{p}_a} \cdot \bar{A} \bar{F}_a + \frac{\partial^2}{\partial \vec{p}_a \partial \vec{p}_a} \cdot \bar{B} \bar{F}_a \quad (3.10)$$

Summarizing the above results, the equation for \bar{F}_a is given by

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \frac{\vec{p}_a}{m_a} \cdot \frac{\partial}{\partial \vec{q}_a} + e_a \vec{E}_a \cdot \frac{\partial}{\partial \vec{p}_a} \right) \bar{F}_a \\ = \left(\frac{\partial F_a}{\partial t} \right)_{\text{coll.}} - \frac{\partial}{\partial \vec{p}_a} \vec{A} \cdot \bar{F}_a + \frac{\partial^2}{\partial \vec{p}_a \partial \vec{p}_a} \vec{B} \cdot \bar{F}_a \end{aligned} \quad (3.11)$$

In order to calculate \vec{A} and \vec{B} , Tchen uses the Fourier integral representation of ϕ_{ab} and F'_{ab} :

$$\begin{aligned} \phi_{ab}(\vec{q}) &= e_a e_b \int_{-\infty}^{+\infty} d\vec{\nu} \exp(i\vec{\nu} \cdot \vec{q}) Y(\nu) \\ F'_{ab}(\vec{q}) &= e_a e_b \int_{-\infty}^{+\infty} d\vec{\nu} \exp(i\vec{\nu} \cdot \vec{q}) Z_{ab}(\nu) \end{aligned} \quad (3.12)$$

Taking Maxwellian distributions for F_a and F_b in the integrands, he investigated \vec{A} and \vec{B} in detail.

IV. EVALUATION OF INTERACTIONS

According to Eq.(3.8), we see that the order of A is

$$O[\vec{A}] = \left(\frac{e^2}{L_D^2} \right)^2 \frac{L_D}{c} \frac{1}{m\bar{c}} \quad (4.1)$$

Considering Eq.(3.11) as the basic equation, we may now examine the relative effects of the terms on the right-hand side. That of \vec{B} is

$$O[\vec{B}] = \left(\frac{e}{L_D}\right)^4 n \frac{L_D}{\bar{c}} = \frac{e^4}{L_D^3 \bar{c}} \quad (4.2)$$

Here \bar{c} is the average speed of particles

$$\bar{c} = (3kT/m)^{1/2} \quad (4.3)$$

and we may take for the order of operator $\partial/\partial \vec{p}$

$$O\left[\frac{\partial}{\partial \vec{p}}\right] = \frac{1}{m\bar{c}} = \frac{1}{(3kTm)^{1/2}} \quad (4.4)$$

Considering

$$L_D^2 = \frac{kT}{4\pi n e^2} \quad (4.5)$$

we obtain for the orders of $\frac{\partial \vec{A} F_a}{\partial \vec{p}}$ and $\frac{\partial^2 \vec{B} F_a}{\partial \vec{p} \partial \vec{p}}$

$$O\left[\frac{\partial}{\partial \vec{p}} \vec{A} F_a\right] = O\left[\frac{\partial^2}{\partial \vec{p}^2} \vec{B} F_a\right] = \frac{e}{(12\pi)^{1/2} m^{1/2} n^{1/2} L_D^6} \quad (4.6)$$

We may estimate the order of $(\partial/\partial t)_{\text{coll.}}$ by

$$O\left[\left(\frac{\partial F_a}{\partial t}\right)_{\text{coll.}}\right] = \pi R^2 \bar{c} n = \frac{e n^{1/2}}{3 \times (12\pi)^{1/2} m^{1/2} L_D^3} \quad (4.7)$$

Making ratios of these values, we have

$$\frac{O\left[(\partial F_a/\partial t)_{\text{coll.}}\right]}{O\left[\partial \vec{A} F_a/\partial \vec{B}\right]} = \frac{O\left[(\partial F_a/\partial t)_{\text{coll.}}\right]}{O\left[\partial^2 \vec{B} F_a/\partial \vec{p}^2\right]} = 4\pi n L_D^3 \quad (4.8)$$

We interpret Eq.(4.8) as follows:

By putting

$$\delta = n^{-1/3} \quad (4.9)$$

we have

$$L_D/\delta = \frac{(kT)^{1/2}}{(4\pi)^{1/2} e n^{1/6}} \quad (4.10)$$

From Eqs. (2.5) and (4.5), we have

$$\frac{L_D}{R} = \frac{\frac{3}{2} \times (kT)^{3/2}}{(4\pi)^{1/2} e^3 n^{1/2}} \quad (4.11)$$

It is easily shown that

$$\frac{L_D}{R} = 4\pi \left(\frac{L_D}{\delta} \right)^3 \quad (4.12)$$

$$\frac{\delta}{R} = \frac{L_D}{R} \frac{\delta}{L_D} = 4\pi \left(\frac{L_D}{\delta} \right)^2 \times \frac{3}{2} = 6\pi \left(\frac{L_D}{\delta} \right)^2 \quad (4.13)$$

or

$$\log \left(\frac{\delta}{R} \right) = \log(6\pi) + 2 \log \left(\frac{L_D}{\delta} \right) \quad (4.13a)$$

If we take $L_D/\delta \geq 1$, it is clear that $\delta/R > 1$. When $L_D/\delta = 1$,

we obtain from Eq. (4.10)

$$(4\pi)^{1/2} e n^{1/6} = (kT)^{1/2} \quad (4.14)$$

or

$$\log_{10} n = 3 \log_{10} T + 5.04 \quad (4.14a)$$

where n is in cm^{-3} and T in $^{\circ}\text{K}$. Equation (4.14a) is plotted in

Fig. 2. If L_D/δ is extremely large, we may neglect completely the effect of microscopic encounters and the equation is of the Vlasov type. When L_D/δ is fairly larger than unity, the interaction by the outer core may be neglected and the equation is of the Boltzmann type. As L_D/δ decreases, the friction and diffusion terms appear as subsidiary ones. Near $L_D/\delta = 1$, the effect of the inner core (collision) and the effect of the outer core (friction and diffusion) may be comparable to each other. As L_D/δ decreases further, the collision term may be negligible and the equation is of the Fokker-Planck type. At the same time, the assumption of binary encounters becomes unplausible. In other words, when the equation of the Fokker-Planck type is plausible.

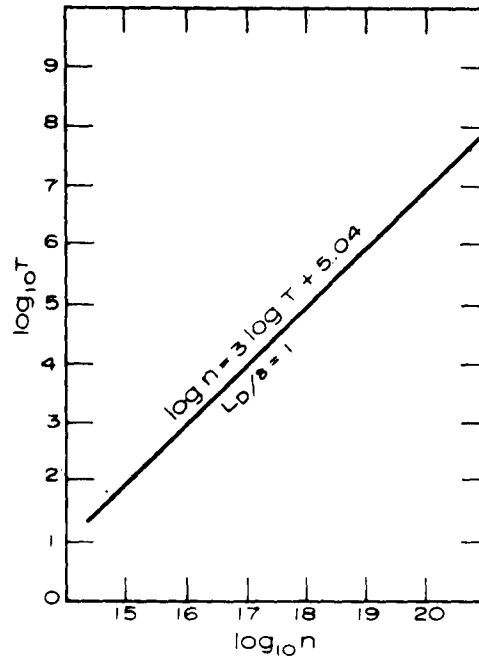


Fig. 2 n is Given By Unit cm^{-3} and T by $^{\circ}\text{K}$ In The Domain Above The Line, $L_D/\delta > 1$.

encounters are to be multiple (Ref. 3). In Fig. 3, these various domains are indicated schematically.

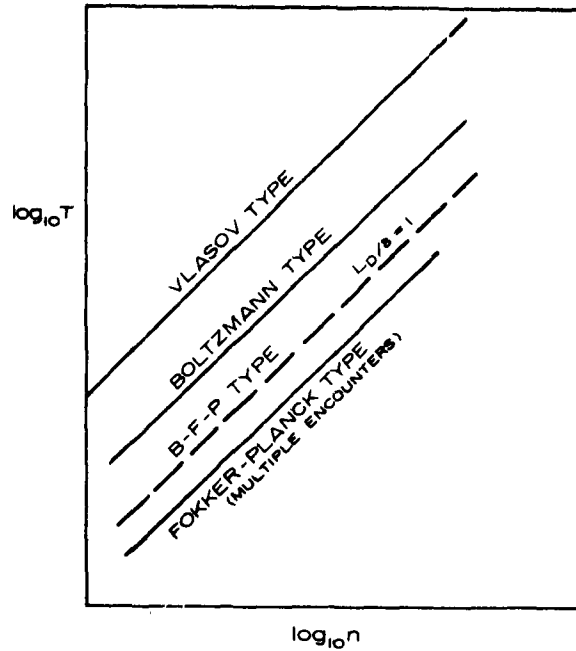


Fig. 3 $\log_{10} T - \log_{10} n$ Space Is Divided By Lines, $L_D/\delta = \text{constant}$, In Four Domains.

The Equation Is:

- (1) Of The Vlasov Type When L_D/δ is Extremely Large;
- (2) Of The Boltzmann Type, as L_D/δ Decreases;
- (3) Of The Boltzmann Type + Fokker-Planck Type (Binary), When L_D/δ Approaches To Unity;
- (4) Of The Fokker-Planck Type (Multiple), As L_D/δ Decreases Further.

In Domain (4), Encounters Are Multiple (More Than Binary).

V. CONCLUDING REMARKS

By the order analysis above, we see that the friction and diffusion terms due to binary interactions have validity only as corrections to the collision term of the Boltzmann equation. Within the limits of binary interaction these Fokker-Planck terms by themselves appear to have no region of validity. When the collision term of the Boltzmann type becomes negligible compared to the long-range interactions, we must consider multiple (more than binary) interactions.

In calculating $\left(\frac{\partial \bar{F}_a}{\partial t}\right)_{\text{coll.}}$ (3.5), a particle which contributes to this term is to have a velocity satisfying the condition that the nearest distance between these colliding particles is shorter than R .

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